Tuning By Numbers - Course Notes

The physical theory and mathematics of just intonation, and how to apply it in the barbershop style

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    Shown with the help of Sound Hypothesis

Part I - Theory

1. Introduction to the harmonic series and Just Intonation

NB: In this course, we do not need to concern ourselves with the fact that every note sung by a human being is made up of lots of sine and cosine waves superimposed on one another (harmonics). We will imagine that each note is represented only by the "fundamental" frequency.

The harmonic series

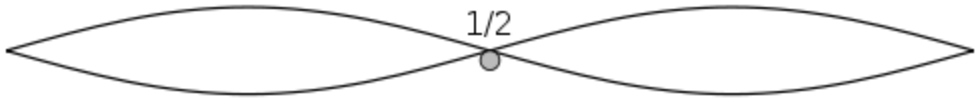
Note: there are lots of good resources online that can provide lots of in depth information about the harmonic series. Here, we only aim for a quick overview.

Definition: (from wikipedia, a bit mathsy)
A harmonic series is the sequence of sounds - pure tones, represented by sinusoidal waves - in which the frequency of each sound is an integer[1] multiple of the fundamental, the lowest frequency.

Imagine a string held still at each end and vibrating:



You can change the frequency of the note by introducing nodes or "null points" in between points 0 and 1. For example:

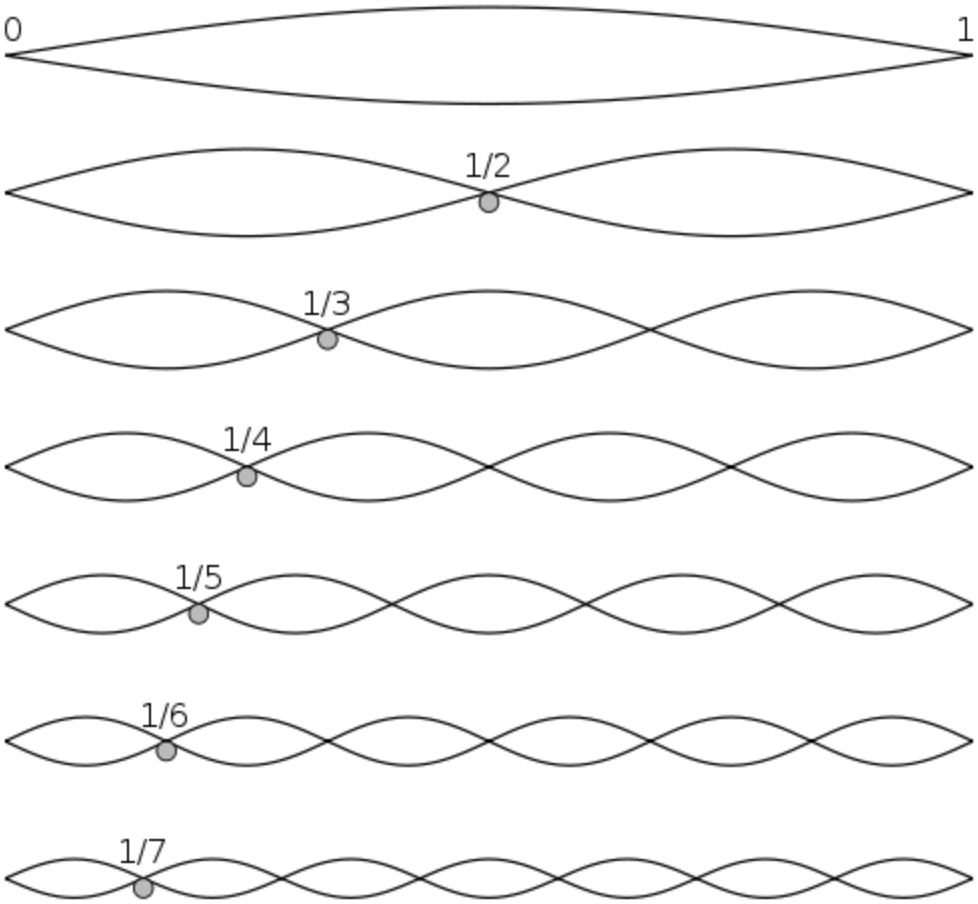


However, you can't introduce the nodes randomly: they must split the string equally into a number of parts, so that the ends are also at 0. For example, this doesn't work:

[1] Remember that an integer is a "whole number" like 1, 2, 17 or 5 (it also includes negative numbers, but we don't need them for this purpose).



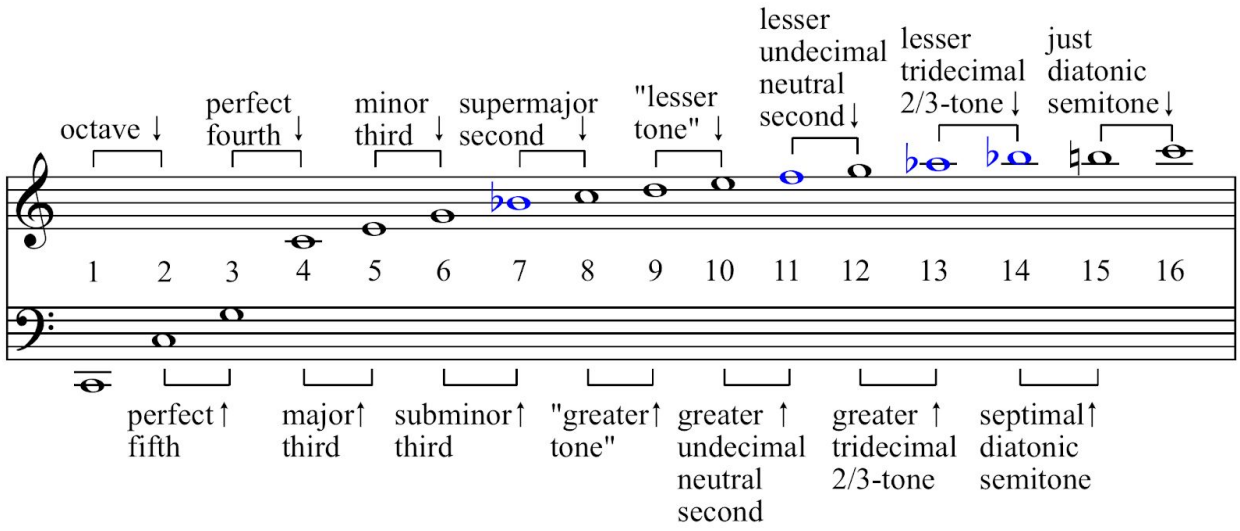
This means you can only get a particular set of wavelengths (and therefore a particular set of frequencies), as you go “up the harmonic series”.



[2]

Each of these “strings” above represents a note with a particular frequency that rises the more nodes we introduce. Since, for example, the difference between 1/7 and 1/6 is less than the difference between 1/3 and 1/4, the notes get closer together as we go up the harmonic series.

[2] http://www.physics.usyd.edu.au/teach_res/hsp/sp/mod31/m31_strings.htm



https://en.wikipedia.org/wiki/User:Hyacinth Blue denotes tones that are a long way from the diatonic scale

We're mostly interested in the first 9 harmonics.[3]

Frequency ratios

I want to take a moment to define what we mean by "frequency ratios", as it is a term we will use throughout. Any note has a frequency in Hertz (or "Hz" for short).[4] The frequencies of E3 and B3, as pictured are 164Hz and 246Hz. I looked them up.

For every two vibrations that the string playing an E makes, the string playing the B makes three. We say that they have a ratio of 3:2 (pronounced "three to two"). We could say 2:3 - as long as we're clear that one of the notes is lower it doesn't matter that much. We could also say

[3] Beyond that, they are mostly either repeats (eg the 10th harmonic is just another major third an octave up from the 5th harmonic) or are highly exotic notes that are very far from the diatonic scale and are hard to ring (eg, the 11th harmonic, which is a justly-tuned #11).

[4] The unit of Hertz means "per second". A skipping rope has a frequency of about 1Hz, the day/night cycle has a frequency of about 0.000012 Hz.

they have a ratio of “1.5”, as $3 \div 2 = 1.5$. This means you have to multiply the frequency of the lower note by 1.5 to get to the frequency of the higher note.

Just Intonation

When we tune intervals using Just Intonation we follow the harmonic series as closely as possible, as much as possible. Now we know what frequency ratios are we can write out the following table.

Interval	Example notes	Frequency ratio
Octave	*C - C*	1 : 2
Perfect Fifth	*C - G*	2 : 3
Perfect Fourth	*G - C*	3 : 4
Major Third	*C - E*	4 : 5
Minor Third	*E - G*	5 : 6
Minor Third (*5th - 7th*)	*G - Bb*	6 : 7
Tone (*7th - 8ve*)	*Bb - C*	7 : 8
Tone (*8ve - 9th*)	*C - D*	8 : 9
Tone (*9th - 3rd*)	*D - E*	9 : 10

Interval	Example notes	Frequency ratio
More exotic intervals		
Major 3rd (*7th - 9th*)	*Bb - D*	7 : 9
Major sixth	*C - A*	3 : 5
Minor sixth	*E - C*	5 : 8
Minor (dominant) seventh	*C - Bb*	4 : 7
Major seventh	*C - B*	8 : 15
Tritone (small: *3rd - 7th*)	*E - Bb*	5 : 7
Tritone (large: *7th - 3rd*)	*Bb - E*	7 : 10
Major 9th	*C - D*	4 : 9

2. Equal Temperament

Introduction

~~Equal temperament is an evil, cunning, trick of the devil to subvert the one true tuning, and should be rejected at every turn...~~

Actually, if barbershop groups - choruses or quartets - sang as in tune as a piano, the international top 20 (and maybe even the top 10) would probably sound a lot better. It's worth remembering that the 12-tone scale came about as a result of observations of the harmonic series.

Equal temperament is a system under which each semitone in an octave is the same as any other. It follows a long line of "temperaments" that emerged throughout the baroque and classical periods. Each temperament is well understood to be a "compromise" that would enable some chords in some keys to sound "perfect", while others must, necessarily, sound less good. A keyboard instrument could be tuned to a given temperament, for a suitable key, before being played. A piano or a harpsichord tuned to equal temperament will sound equally "fine" in any key. In contemporary music, temperaments other than *Equal Temperament* are rarely used.[5]

As singers we are not bound to 12 pitches that must be determined before the start of the concert. In barbershop harmony, we work on a system known as Just Intonation. It is worth noting, however, that *just intonation* is far from without compromises! For the purposes of ringing chords, however, the benefits generally outweigh the drawbacks. As you may have guessed, Just Intonation uses the relationships we looked at in the harmonic series.

Why don't Equal Temperament and Just Intonation agree with each other?

We're going to look at a (thorough) mathematical explanation of why *Equal Temperament* and *Just Intonation* don't agree. We need three additional definitions before we start.

1) There are twelve semitones in an octave

2) Let $f_1,\ f_2, f_3 \ldots f_{13}$ be the frequencies of 13 notes each separated by a semitone (eg *C* to *C)*

3) The frequency ratio of two notes an octave apart is $2:1 \ \ or \ \ 2/1 = 2$

[5] It's a common myth that Bach came up with Equal Temperament. He didn't (Equal Temperament was not invented until the 20th century). Among many others, he used "well temperament", which facilitates playing in every key. In well temperament all keys have different "characters" on account of different size semitones in the octave.

Let's get started!

$$\frac{f_{13}}{f_1} = 2 \qquad \textit{(equation 1)}$$

This comes directly from the third definition. We can pull off a trick here, and rewrite equation 1 as the below, because the additional terms all **cancel out.**[6]

$$\frac{f_2}{f_1} \times \frac{f_3}{f_2} \times \frac{f_4}{f_3} \times \ \ldots \ \times \frac{f_{13}}{f_{12}} = 2 \qquad \textit{(equation 1a)}$$

According to the original premise of *Equal Temperament: "*every semitone must sound the same".
What that means mathematically is:

$$\frac{f_2}{f_1} = \frac{f_3}{f_2} = \frac{f_4}{f_3} \ \ldots \qquad \textit{(equation 2)}$$

$$\left(\frac{f_2}{f_1}\right)^{12} = 2 \qquad \textit{(combining equation 1a and equation 2)}$$

$$\frac{f_2}{f_1} = \sqrt[12]{2}$$

$$\frac{f_2}{f_1} = 1.0594630944$$

$$\frac{f_2}{f_1} \approx 1.06$$

We can now make a formal *definition* of a semitone. The semitone is defined as the ratio of f_2 to f_1 **,** which is an actual number (see **Frequency ratios,** above, for more information).

Let $s = \frac{f_2}{f_1} \approx 1.06$

[6] It's easier to see how this is true if you try to show that equation 1a can be turned into equation 1. f_2 on the top on the first term cancels with f_2 on the bottom of the next term, f_3 on the top cancels with f_3 on the bottom of the next term, etc. The only two terms without a partner to cancel are f_1 and f_{13}.

A perfect fifth (in *Equal Temperament*) is 7 semitones. "Going up 7 semitones" means multiplying s by itself 7 times. (If this "multiplying" seems unintuitive, recall that to "go up an octave" requires multiplying the frequency by 2 (doubling it).)

$$s^7 = 1.4983070769$$

A perfect fifth (in *just intonation*) is a ratio of $2:3 \quad or \quad 3/2 = 1.5$

So close! Yet so far. Seven equal tempered semitones are *not quite* equal to a true perfect 5th.

$$s^7 \neq 1.5$$
$$1.498... \neq 1.5$$

How can we fix this?

We've shown why a perfect fifth on a piano in *Equal Temperament* is "wrong". What adjustments do we have to make to fix this?

A "cent" is one 100th of a semitone.

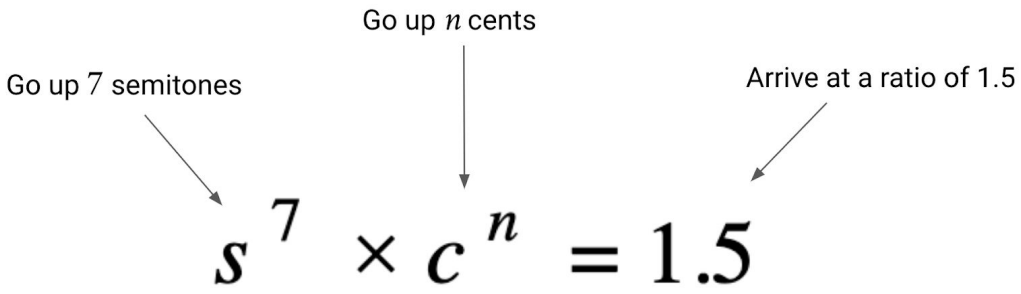
In equation form:

$$1\ cent = c = \sqrt[100]{s} = 1.0005777895$$

Example 1 - perfect 5th

By how many cents do we need to adjust to arrive at *exactly* 1.5? In other words, what is the value of n? In the same way that "going up a semitone" means we must multiply by s, "going up a cent" means we must multiply by c (this is the way we've defined both c and s, above)

This means we can write the following equation:



$$c^{n} = 1.5/s^{7}$$

$$n = log_{c}(1.5/s^{7})$$

$$n = 1.955...$$

$$n \simeq 2$$

Let's do a sanity check:

$$s^{7} \times c \times c = ?$$

$$1.4983 \times 1.0005777895 \times 1.0005777895 = 1.5000...$$

Great!

Example 2 - major 3rd

How many cents do we have to move from a piano major 3rd to reach a justly intonated major 3rd? Easy! Instead of $3:2$, we use $5:4$ *or* $5/4 = 1.25$ because this is the *just intonation* ratio of a major third. Instead of s^{7} we use s^{4} because in *Equal Temperament* a major 3rd is 4 semitones. We can jump in halfway through the calculation from above, with the new numbers.

$$s^{4} \times c^{n} = 1.25$$

$$n = log_{c}(1.25/s^{4})$$

$$n = -13.69...$$

$$n \simeq -14$$

Note the spontaneous appearance of the minus sign (a great feature of logarithms!). While a perfect fifth needs to be *widened* by 2 cents a major third needs to be *narrowed* by 14 cents. In general, "$+$" means the interval must be ***widened***, and "$-$" means the interval must be ***narrowed***. Sometimes the upper note will move and sometimes the lower note may move, meaning a $+$ could mean raising the upper note ***or*** flattening the lower note (or a mixture of both).

Table of cent adjustments for common barbershop intervals

Interval	Example notes	Frequency ratio	Adjustment to achieve Just Intonation (cents)
Octave	*C - C*	1 : 2	0
Perfect fifth	*C - G*	2 : 3	2
Perfect fourth	*G - C*	3 : 4	− 2
Major third	*C - E*	4 : 5	− 14
Minor third	*E - G*	5 : 6	16
Minor third (*5th - 7th*)	*G - Bb*	6 : 7	− 29
Tone (*7th - 8ve*)	*Bb - C*	7 : 8	31
Tone (*8ve - 9th*)	*C - D*	8 : 9	4
Tone (*9th - 3rd*)	*D - E*	9 : 10	− 18

Interval	Example notes	Frequency ratio	Adjustment to achieve Just Intonation (cents)
More exotic intervals			
Major third (*7th - 9th*)	*Bb - D*	7 : 9	35
Major sixth	*C - A*	3 : 5	− 16
Minor sixth	*E - C*	5 : 8	14
Minor (dominant) seventh	*C - Bb*	4 : 7	− 31
Major seventh	*C - B*	8 : 15	12
Tritone (small: *3rd - 7th*)	*E - Bb*	5 : 7	− 17
Tritone (large: *7th - 3rd*)	*Bb - E*	7 : 10	17
Major 9th	*C - D*	4 : 9	4

3. Justly tuning whole chords

So far we have looked at tuning intervals according to the harmonic series. That's great, and all, but how does this apply to chords? Can we just tune every interval according to the table above? Well yes...and no.

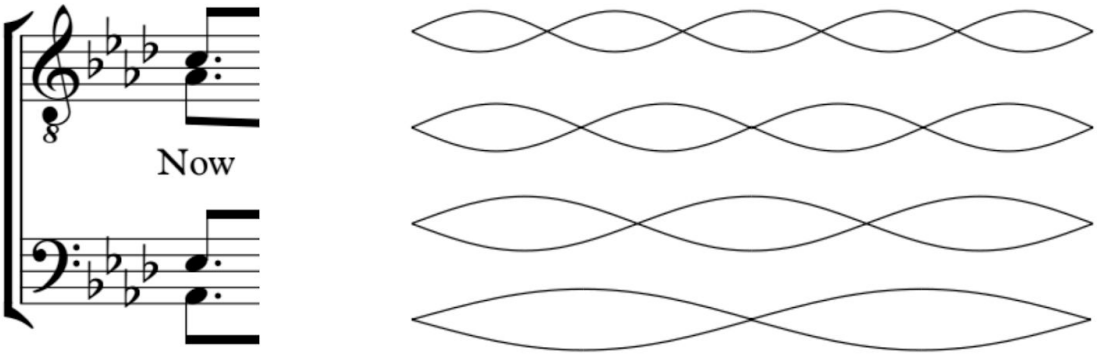
The objective of justly tuning chords

The "aim of the game" can be summed up as follows: tune the notes such that they share integer relationships that are as *simple* as possible. In this context, "simple" means, for example, 2:3, rather than, say, 22:31. Both of those represent fifths, of sorts, and both of them have integer ratios. The first one is simpler, however, and will inevitably sound better.

What this means in practice is that you can't *always* look up your tuning from the table, tune every part with every other and expect it to work. 4ths and 5ths are safe, but 3rds and 6ths, 7ths, and tones are not, as there are multiple ratios (and tunings) to choose from (see tables above - there are two entries for "major third", relating to different ratios). Instead, we must take into account the whole chord, and seek to make the ratios as "simple" as possible.

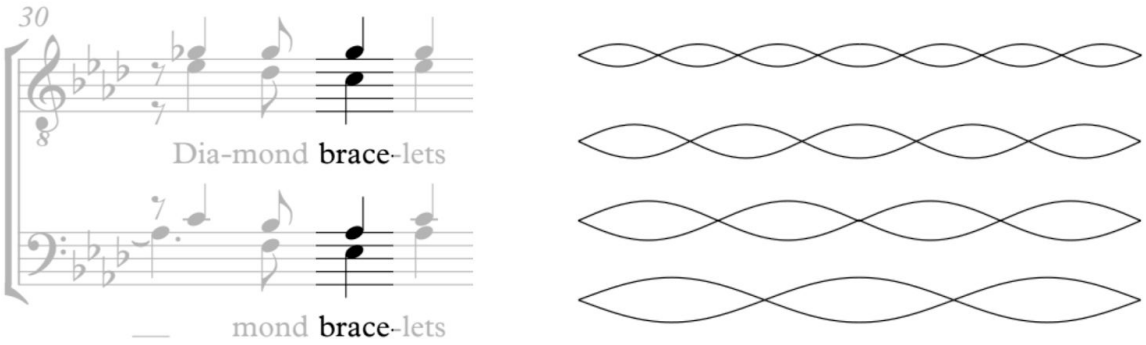
Integer ratios for common barbershop chords

A major chord, voiced "1-5-1-3" can be tuned so that its notes have frequency ratios 2:3:4:5. That's a great ratio!



An Ab major chord, with ratio 2:3:4:5

A dominant 7 chord, voiced (1-3-5-7) can be tuned so its notes have ratios 4:5:6:7.
Or if it was voiced 5-1-3-7 that would be 3:4:5:7. Even better!
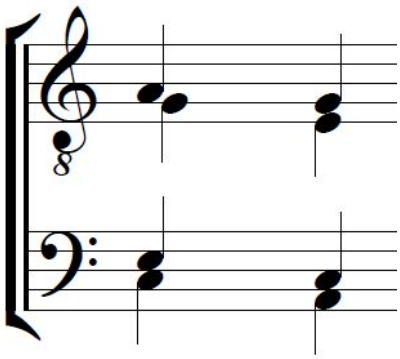


An Ab dominant 7 chord, with ratio 3:4:5:7

We could go through all of the chords in different voicings, but it would take a while. Simon Rylander, now the baritone in Stockholm Syndrome, prepared this excellent resource back in 2008. “The 11 Chords of Barbershop” - https://www.sunshinetracks.com/chords.pdf - is highly recommended reading.

Alternative tunings of maj 6 and minor 7 chords

There are two chords in barbershop that have the same notes. These are the major 6 chord and the minor 7 chord.



Shown above are a C major 6 chord and an A minor 7 chord. Both contain the notes (C, E, G, A)

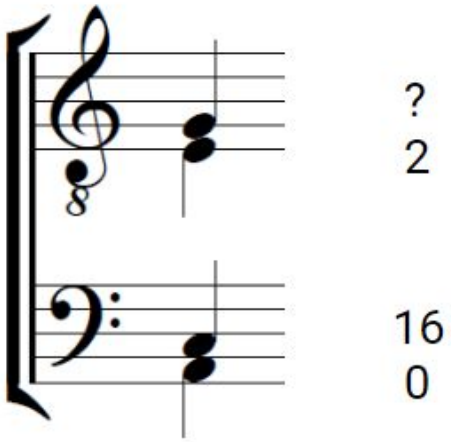
However, this does not mean that the two chords are generally tuned in the same way! Let’s explore this a bit further.

Let's voice both chords as a minor 7 to make life simple[7]. (A, C, E, G).

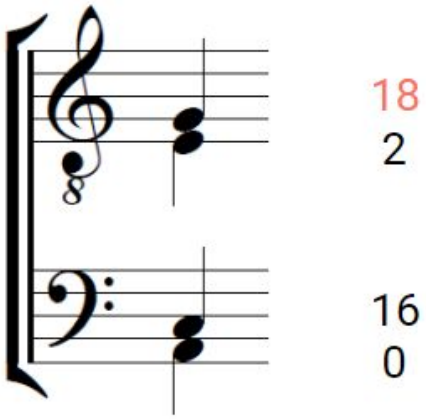
Observations:

- The A and E are a 5th apart, and so too are the C and G.
- The first three notes make a minor triad.
- There's a 7th between the A and G.

If we tune the minor triad with a traditional, large, minor third[8], we get tunings of:



How should we tune the G? Well, it's a fifth away from the C, so we should keep that interval true. So then we have:



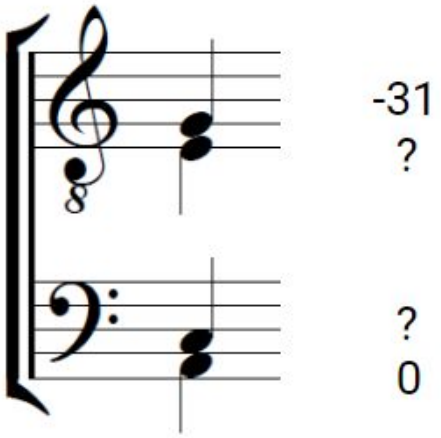
Sounds great! There's a justly tuned major third, and a couple of fifths.

Alternatively, we could consider that the dominant 7 should be our starting point, rather than the minor third. After all, is the 7th not the most important degree of the scale for barbershoppers?

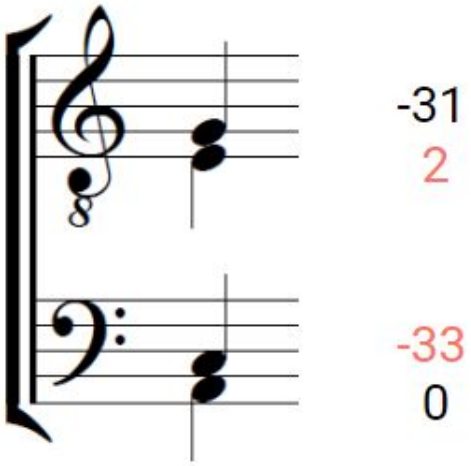
[7] While you frequently might choose to tune a min7 as a maj6, you would rarely want to tune a maj6 as a min7.

[8] As found in the tables above (simple minor third - ratio 5:6).

In this instance, we start with the following tunings:



How should we tune the E and C ? Easy: they’re fifths away from the A and G respectively.



The ratios are as follows:

Major 6 tuning - 10 : 12 : 15 : 18 - (A, C, E, G)
Minor 7 tuning - 12 : 14 : 18 : 21 - (A, C, E, G)

These are both quite complex ratios.

The half-diminished / min 6 / rootless 9 chord

The half-diminished chord is my all-time favourite chord in barbershop. As well as being a bread-and-butter chord, it is also often found at climactic moments across our style. For example:

*That Old Quartet of Mine: "just to sing that song just one more **time**"*
*I Won't Send Roses: "still while there's a fighting chance, just turn and **go**"*
*She's Out of My Life: "My **life!**"*
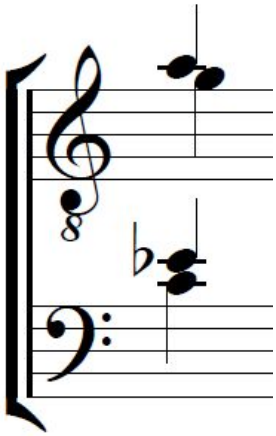


Instant Classic and Boston Common at the moment of (musical) climax

The optimum ratio (when voiced 3-5-7-9) is $5:6:7:9$. I mean, wow! Compare that to the major 6th ($12:15:18:20$) and you can quickly see it is a *very simple* ratio. It rings great!

You might find that all this talk of ratios is quite abstract. How do we tune the chord, compared to a piano? The exercise below might help us find out.

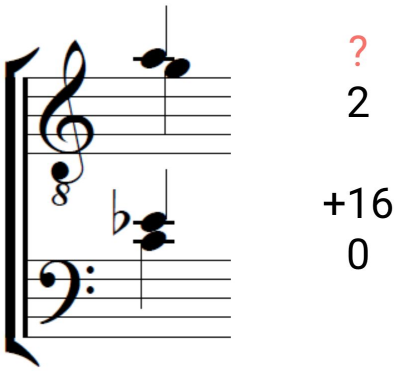
Let's take the chord to be: C,Eb,G,A.



Observations:

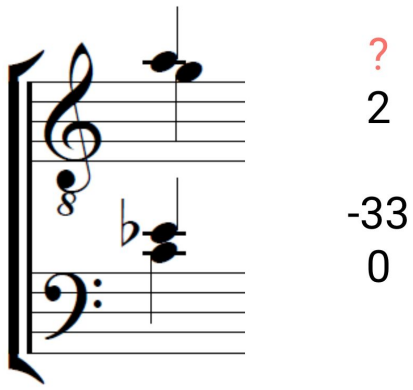
- Perfect 5th between C and G.
- Tritone between Eb and A.
- First three notes make a minor triad.

Let's try the same approach we took before - start with the minor triad.
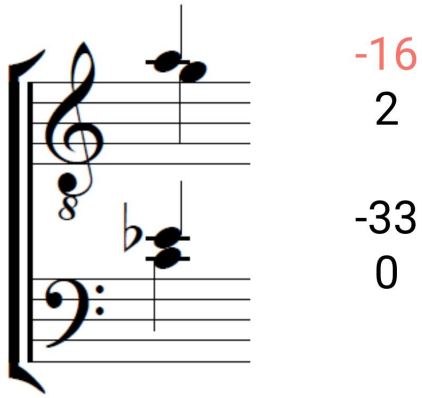


So how do we tune the A. Well, it's in a tritone with the Eb...but should it be a big tritone ($+17$) or a small one (-17)? It's not clear. Either way, that's going to result in a pretty weird tone from G to A. Is there an alternative?

Remember before, when we chanced upon an alternative minor triad tuning.[9] The cent adjustments associated with it were:
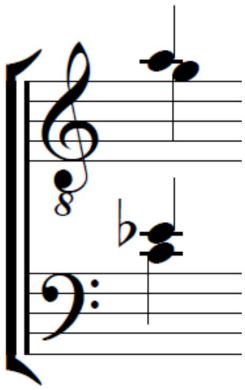


Could this be useful? A wide tritone from Eb to A gives us:



[9] Note that this utilises the “other” minor third (and major third) from the tables above with a ratio of 6:7 (and 7:9).

Is this any better? All the numbers still look a little unfamiliar. Here comes the trick. We're going to move up all the cent tunings by 2 cents.
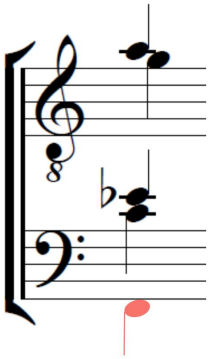


-14
4

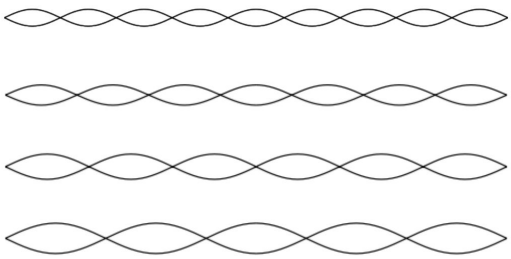
-31
2

Do any of these numbers look more familiar?
$+2$ is the tuning for a perfect 5th
-31 is the detuning for a 7th.
$+4$ is the tuning for a 9th.
-14 is the detuning for a major third.

What if we imagine an extra note on the bottom - an F. All of a sudden, the chord makes sense as a *rootless* F chord, with a 5th (the C), 7th (the Eb) a 9th (the G) and a major 3rd (the A).



-14	3rd
4	9th
-31	7th
2	5th
0	TONIC!

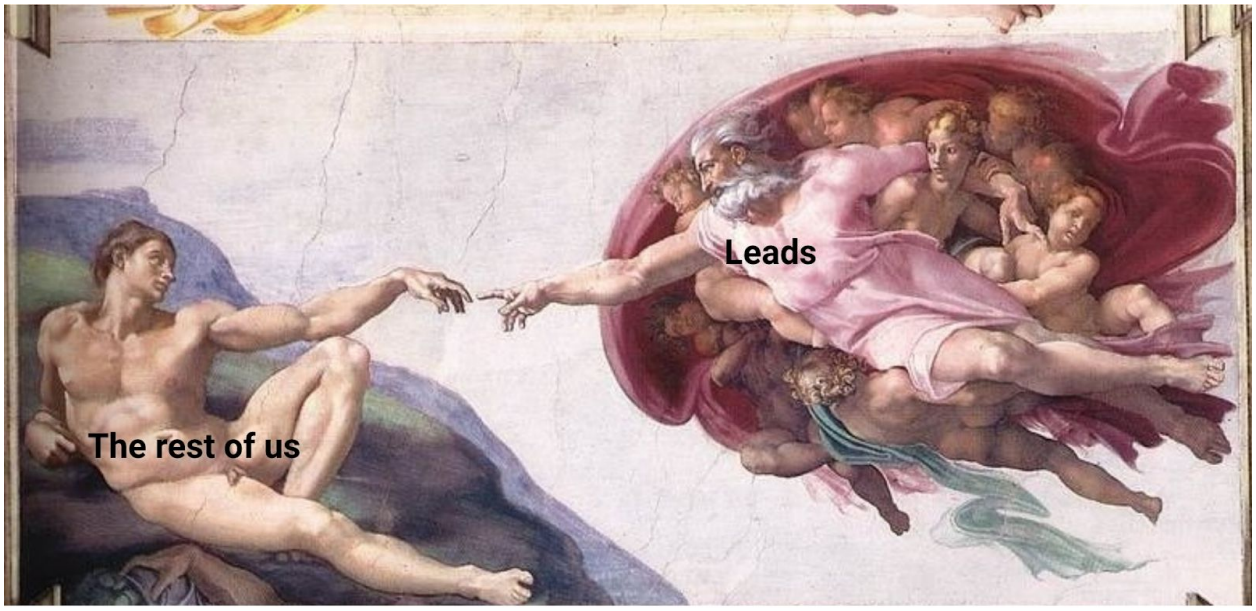


It turns out that this cent adjustment is the best we can do, in order to get the simplest ratio for this voicing ($6:7:9:10$).

Part II - Putting it into practice

4. Tuning around the lead

"Lead is God"



An accepted principle of tuning in barbershop is that the harmony parts tune around the lead. What do we mean by "tune around" the lead? It means that lead should sing the melody, unhampered by just intonation considerations, while the other three parts should make adjustments to accommodate him or her in the chord, such that the relationships we've been discussing above are met.

There are various schools of thought about how the melody should be tuned[10], but as a starting point, let's imagine that the lead sings perfectly in tune with a piano - in other words he or she is making an adjustment of ZERO cents on each note.

[10] Some people make an argument for the lead using Pythagorean tuning. It seems to me that this can create huge problems to be overcome, and so I suspect that this is not as prevalent as some might think.

“Flattening” and “Hoiking”.

In Sound Hypothesis we use the terms “flattening” and “hoiking” to refer to the adjustment required by the harmony parts to accommodate the lead in a chord. For example: if a lead sings an E in a C major chord, then he or she is singing the third. If the E is to be sung with no adjustment from the piano (0 cents) then the other three parts must first adjust upward (“hoik”), by an amount equal to the *downward* adjustment that a major third normally has to fit within a chord (14 cents), in addition to making their own adjustments based on their role in the chord. For example, in a chord voiced like this:

G, C, E, Bb (C7, second inversion)

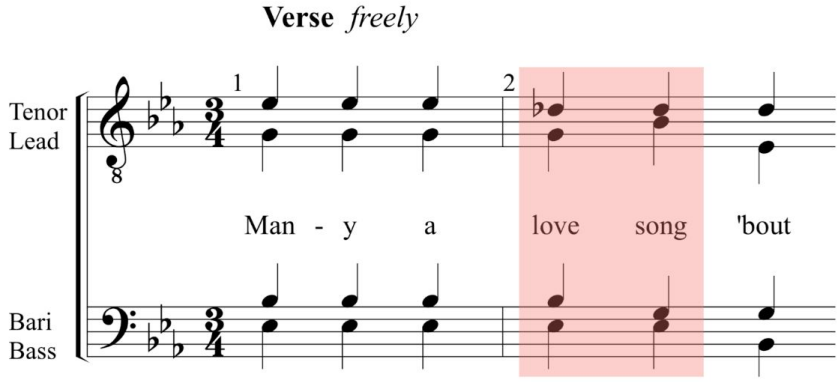
The tenor must make an adjustment of +14, to accommodate the fact that the lead is on the major 3rd, in addition to an adjustment of -31, because they are the 7th. This results in an overall tuning of -17. The bass and baritone end up at +16, and +14, respectively. We’ll see a general “step-by-step” guide to this later.

Exceptions to the rule

There are, however, occasions when it doesn’t make sense for the harmony parts to adjust, or at least, not rigidly according to the maths presented above. There are people who will claim that you must **always** tune around the lead. I do not believe this is true, and so below we visit a few scenarios where alternative ideas are appropriate.

Exception 1: Voice swapping

Two parts, eg baritone and lead, swap notes in a chord, while the other parts stay on the same notes.



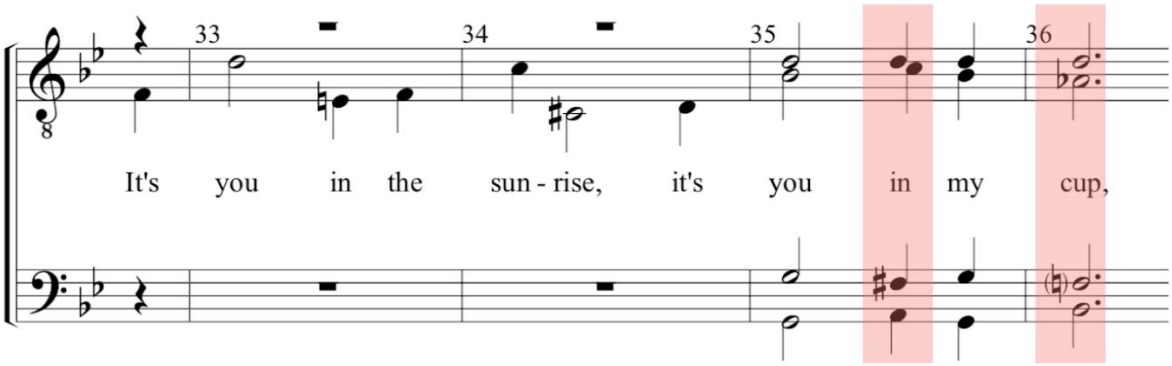
“It’s You” - arranged by Robert Rund

In the above example, the tonal centre is established by the first four notes. Since two parts (bass and tenor) remain the same on the words “love song”, and the chord does not change, it makes little sense to start reasserting the tonal centre from one chord to the next. This would be jarring for the audience at best, and dangerous for the singers at worst. It is far better to have the baritone and lead trade notes exactly. This means that “Lead is God” has been broken.

Exception 2: Extreme adjustment

There are times when following “Lead is God” would require *extreme adjustment* to be made by the harmony parts. One typical example is when the lead is on a 7th. The best quartets manage to make this sound effortless and natural, and I’m still not sure how they do it! I suspect that it’s a combination of a few things:

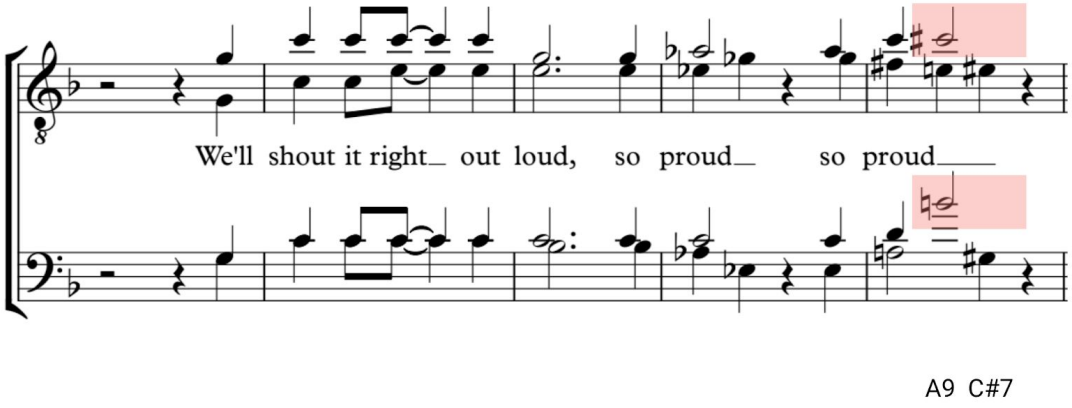
- **The 7th is not tuned as a true 7th.** The 7th is actually very forgiving. Depending on things such as the volume, the flavour of the chord and whether or not it’s a melody note, the 7th has a large range of acceptable tunings (including one where the 7th is at +15, in a minor third with the 5th).
- **The lead “accommodates” the harmony parts**. The parts can both adjust their tuning slightly and “meet in the middle” - a compromise, if you will.



*“It’s You”. On the word “cup” the lead is on an Ab, the **7th** of the chord (Bb) on the word “cup”. Compromise is the best option, to avoid “extreme adjustment” from the harmony parts. On the word “In” the lead is on the 7th as well, but more importantly the tenor is on a steady D. We chose to keep the tenor constant through “you in my” and have him move slightly on “cup”.*

Exception 3: Posts or pedals (held notes)
This exception is obvious (or is it?) At the end of a song, a tenor post provides a perfect example of when a quartet should not tune around the lead - instead, we keep the post steady, and make adjustments accordingly (hoiking or flattening). There is a striking example of this in *Hello My Baby - Ringmasters tag. See section 6*.

However, there are occasions where a steady note may not be the best note to tune around. A great example is the "middle tag" of *Yes Sir That's My Baby,* as performed by Ringmasters.



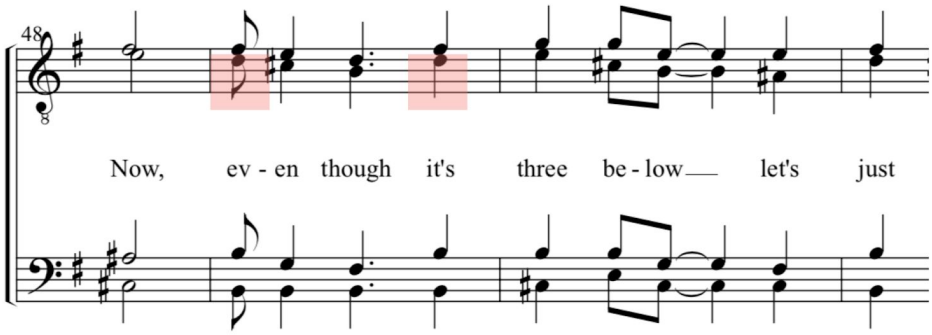
This is absolutely loaded with tuning anomalies! For now, let's look at the final swipe.

- The penultimate chord is an A9, while the final chord is a C#7
- the B♮ in the baritone is a 9th, and then a 7th
- the tenor C# is a 3rd, then a tonic

As you will know by now, a change in the relationship between the baritone and tenor will result in microtuning changes as the swipe occurs, even though the written notes do not change. The question is, which note should we keep steady? Which note should fulfil the role of "post"? If we choose the baritone B, this would require a huge hoik from the other three parts. In particular, a hoik of 49 cents (half a semitone!) would be required from the tenor. If we choose the C#, we risk lowering the tonal centre of the piece, as the 3rd becomes a tonic. In theory, the tonal centre would sag by 14 cents. The solution Sound Hypothesis came to was to move **both** notes slightly apart from one another, taking the bass as "piano" for those two notes.

Exception 4: A feeling of going flat

A feeling of going flat is invariably followed by actually going flat, or worse, a divergence of "tuning opinions". There are rare occasions when tuning around the lead will lead to a feeling of flattening - typically if the lead is on the minor third. In minor chords, Sound Hypothesis often employ a tactic of holding the three parts steady while the lead makes an adjustment.
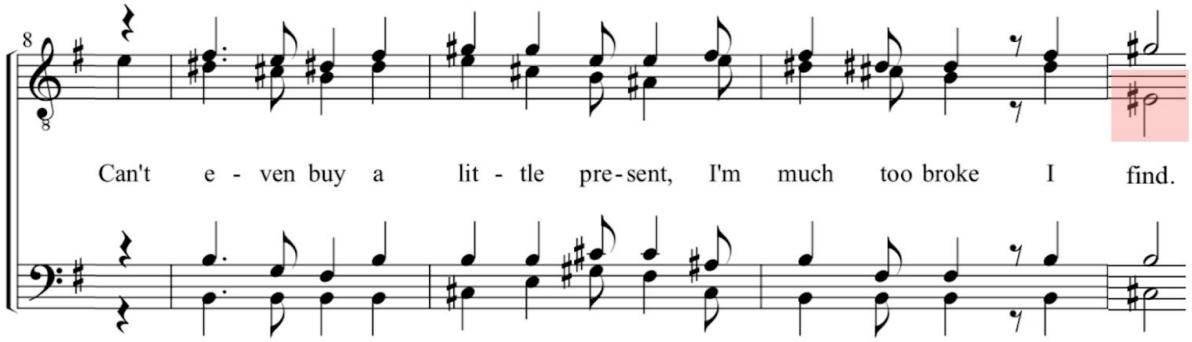


I Can't Give You Anything But Love - David Harrington - minor thirds in the Lead

Exception 5: Low triads where the lead is on the 3rd

Chords with a low triad tend not to ring particularly well[11]. We have found that when the lead is on the third, a chord will sound more lively if the third is not justly tuned. If it is sung with just intonation the third can sound flat. This is the only scenario we have found where just intonation is not desirable. Note that some people believe that a chinese 7th (a 7th chord voiced 5-3-7-1, or 3-5-7-1) should not be tuned with just intonation and that the 7th should be sharp. On balance, we disagree.

In the below example, the chord of interest is "find". We suggest that the chord sounds "fresher" if all parts except baritone sing in Equal Temperament, resulting in a "high" third in the Lead.

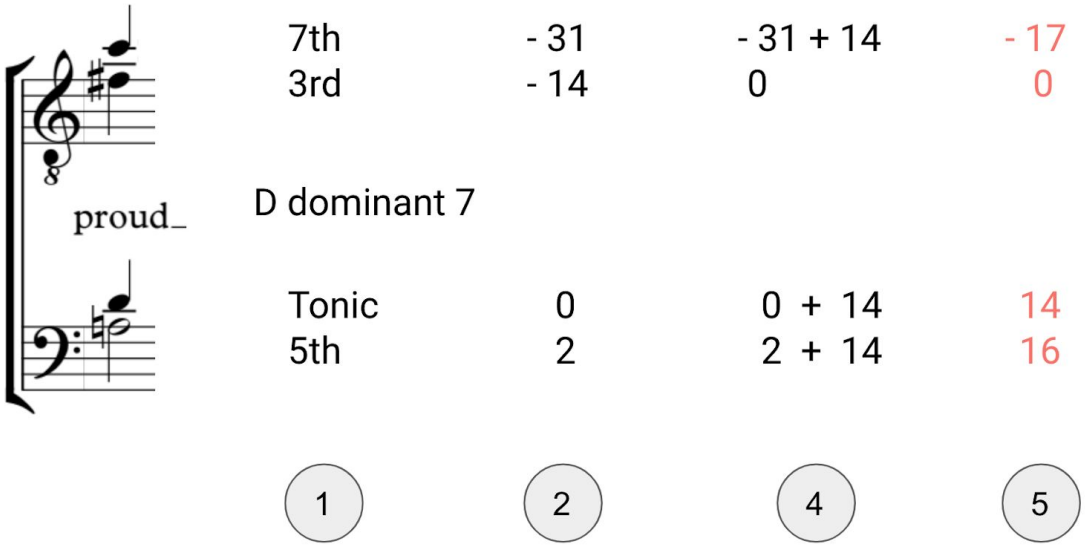


"I Can't Give You Anything But Love" - Arr. David Harrington

[11] A possible explanation is the "lower interval limit".
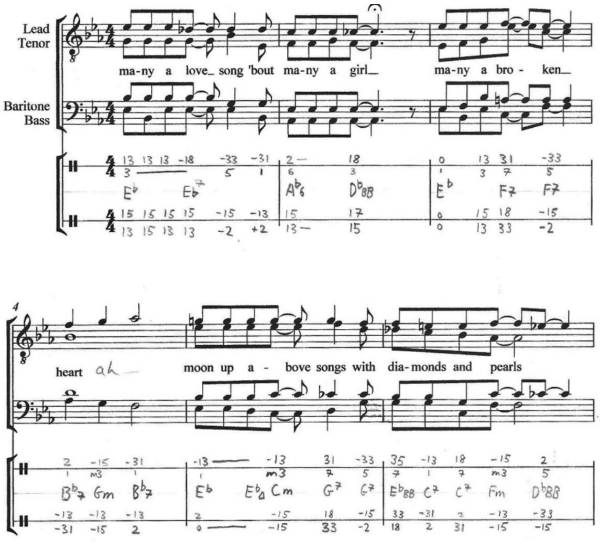
5. Step-by-step: Annotating cent adjustments

Step 1: Identify the chord. The hardest one to spot is the rootless 9, because it's missing the tonic.
Step 2: Apply cent adjustments to each note (without annotating yet) by noting what role they fill in the chord (eg major third is adjusted by -14 cents)
Step 3: Decide which note to designate the "zero note". This is normally the lead, but could be other parts - eg a post.
Step 4: Subtract away the cent adjustment of the "zero note" from all other notes. This will leave the relationships between notes unchanged.
Step 5: Write in the resultant cent adjustments



The various steps. For step 3, we choose the lead as the "zero note".

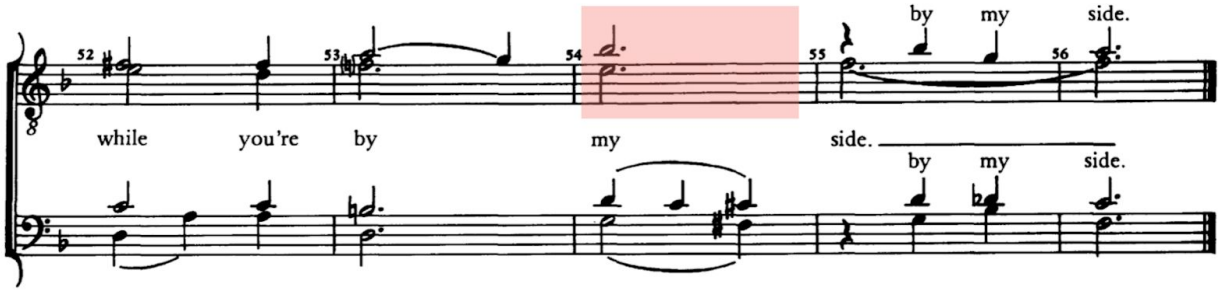
It is possible to take the obsession a bit too far…



6. Real life examples

Some more examples from songs, shown with the help of [Sound Hypothesis]

Sweet and Lovely - Moving across the clock



C9 C7 F#7

The "clock" method is not commonly used these days, in part due to the more exotic progressions that have become normal in barbershop. It involves setting out the circle of 5ths in clock formation. Chord progressions that move up or down a 5th travel one "hour" round the circle. Progressions that move a tritone travel as far away on the clock as it is possible to go. For example, while C is at 12 o'clock, F# is at six o'clock. Dominant 7 chords a tritone apart share two notes - but their roles in the chord *swap*. In the above example, the tenor begins "my" as the 7th (C dom7) and ends as the 3rd (F# dom 7). This is known as moving "across the clock".[12]

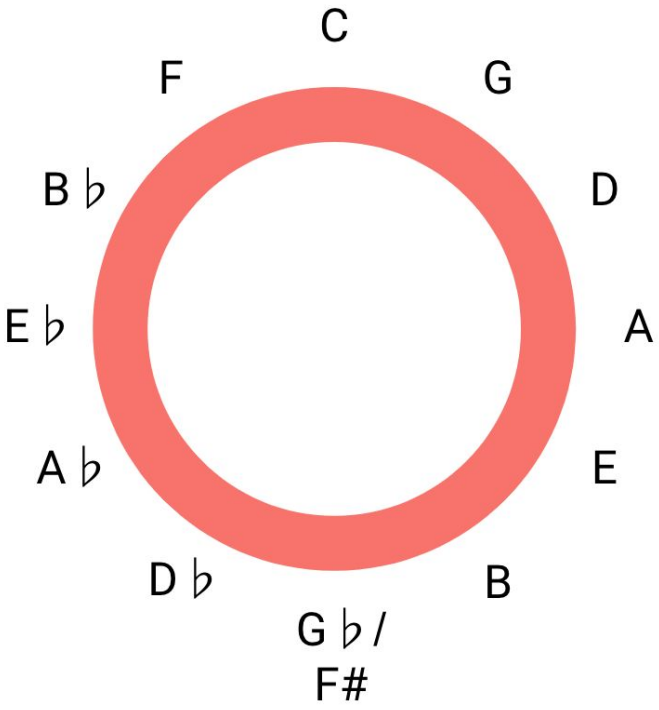
We saw earlier that, since 3rds and 7ths are tuned[13] differently, there are two sizes of tritone that occur in the harmonic series.

Interval	Example notes	Frequency ratio	Adjustment (cents)
Tritone (small: *3rd - 7th*)	*E - Bb*	5 : 7	− 17
Tritone (large: *7th - 3rd*)	*Bb - E*	7 : 10	17

To tune the two chords correctly, the tritone must *grow* by 34 cents between the two chords. In this particular instance, since the lead is still on the melody, the best option is for the *tenor* to make the change. Of course, the baritone and bass must also remember to "hoik" to accommodate the lead.

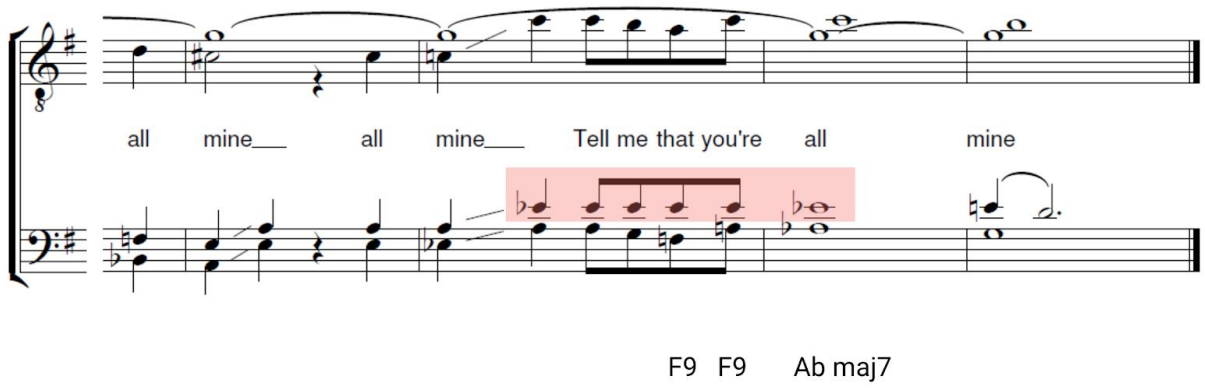
[12] For some tangential, but compelling, viewing, see *What are we trying to preserve? - Dave Stevens.*

[13] that is to say, the adjustments required from equal temperament to match just intonation are -14 cents and -31 respectively



The circle of fifths

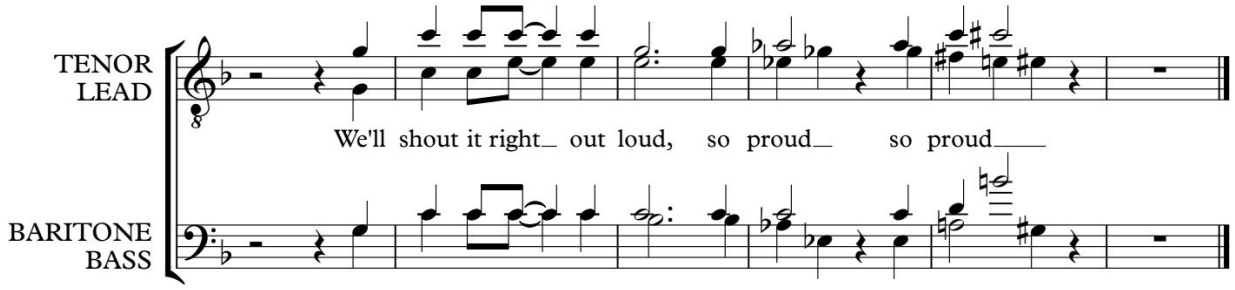
Hello My Baby (Ringmasters tag) - Moving around a post



Interval	Example notes	Frequency ratio	Adjustment (cents)
Major third (tonic to 3rd)	*C - E*	4 : 5	− 14
Major third (*7th - 9th*)	*Bb - D*	7 : 9	35

In this example, the tenor is holding a post on a G. All parts should tune around the post. The notes of interest to us are the Ebs in the baritone. The chords on “That you’re” are both F9 chords - making the baritone a 7th. This means the major 3rd is the **large** major 3rd. The chord on “all”, however, is an Ab major 7, with the post as the major 7th. A major 7th is made up of two perfect 5ths; one between tonic and 5th and one between major 3rd and major 7th. The major third between 5th and major 7 is the **small** major 3rd. Once again, that means someone has to move! In this case, it’s the baritone - a whopping 49 cents between the chords on “you’re” and “all”. That’s nearly half a semitone!

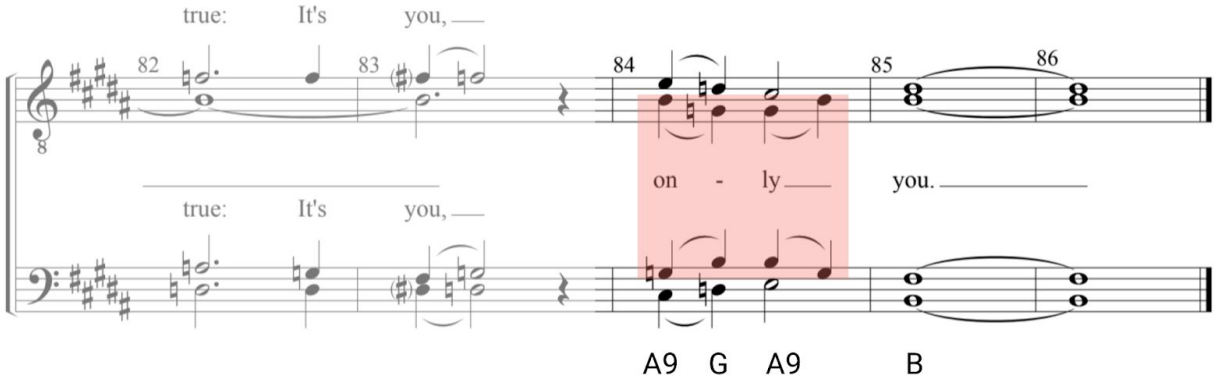
Yes Sir That’s My Baby (Ringmasters alteration) - all sorts going on



We’ve already covered the final two chords of this example in Section 4. However, this short snippet is loaded with opportunities for intelligent tuning decisions. I’ll tell you all the choices we made in Sound Hypothesis, and why.

- On the chord “right”, we have our first choice. Should we designate the lead as “zero” note? This would require all three harmony parts to hoik, despite not changing notes. It seems smart to leave our tonal centre alone and latch on to the bass and baritone as zero notes.
- On the word “loud”, we decided to keep the tonal centre steady again, and allow the bass to adjust his 7th accordingly.
- The biggest choice so far is on the first “proud”. We decided to keep the baritone’s note steady, and have other parts hoik around him. This actually raises the tonal centre. This was not the only choice - we could have chosen the lead - but we enjoyed the “fresher” tone - particularly when the lead swipes to the Gb (7th).
- On the second “proud” we designated bass as zero note.
- The final swipe is discussed in section 4.

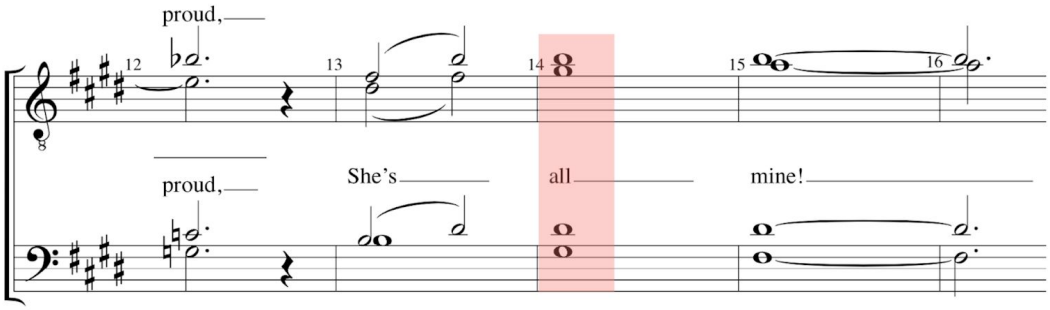
It's You - voice swapping between different tunings



Aha! This looks like a classic voice swapping scenario. You remember what I said about tuning voice swaps: I said it's a case where "Lead is God" is broken. Simply swap the frequencies exactly. In fact, this is a slightly different case, because the tenor and bass **do not stay on the same notes**. In the baritone and lead, the B fills the role of 9th and 3rd, while the G fills the role of 7th and tonic. Have fun!

Yes Sir That's My Baby - Lead is God to the rescue

Typically, keeping a pair of notes steady in this scenario is a good idea. However, in the below example Sound Hypothesis found this led to a "feeling of going flat" on the word "all". We applied another approach. We DO tune around the lead, which means the tenor and baritone sing "all" higher than the second chord of "she's". In this case, "Lead is God" has come to the rescue.



"Yes Sir That's My Baby" - Arr. David Wright - (intro)

Thanks!

If you enjoyed this or, more importantly, if you have any ideas on how I could improve it, please let me know at alexmoore_wcs@hotmail.com . If you reeeally enjoyed it you can buy me a beer! https://paypal.me/alexwillmoore . That would be super cool!